Example 1

1. Suppose an instrument orbiting at 500 km altitude is viewing a dark ocean scene towards nadir in which the radiance is 50 W m^{-2} sr^{-1} μm^{-1} at 0.5 μm. The instrument's telescope focusses an 10×10 cm area on the primary mirror down to a 1000x1000 element CCD detector. The telescope optics gives a pixel size of 250 m on the Earth. There is a 100 nm spectral filter at the 0.5 μm wavelength before the CCD detector. How many photons does each CCD pixel accumulate in a 10 ms exposure?

First, how much energy goes into one CCD pixel during exposure?

$$E = I_\lambda \, \Delta\lambda \, A \, \Omega \, \Delta t$$

Solid angle:

$$\Omega = (\Delta x/R)^2 \;= (250\,\mathrm{m}/500,000\,\mathrm{m})^2 = 2.5 \times 10^{-7}\ \mathrm{sr}$$

where Δx is resolution on Earth and R is distance to Earth.
Collecting area for one CCD pixel:

$$A = \left(\frac{10\ \mathrm{cm}}{1000}\right)^2 = 10^{-8}\ \mathrm{m}^2$$

Energy on one CCD pixel during exposure is:

$$E = (50\ \mathrm{W\ m^{-2}\ sr^{-1}} \mu\mathrm{m}^{-1})(2.5 \times 10^{-7}\ \mathrm{sr})(10^{-8}\ \mathrm{m}^2)(0.100\ \mu\mathrm{m})(0.01\ \mathrm{s})$$

$$E = 1.25 \times 10^{-16}\ \mathrm{J}$$

Second, what is the energy of a photon at $0.5\ \mu$m:

$$E_{photon} = \frac{hc}{\lambda} \;= \frac{(6.626 \times 10^{-34}\ \mathrm{J\ s})(2.998 \times 10^{8}\ \mathrm{m/s})}{0.5 \times 10^{-6}\ \mathrm{m}} = 3.97 \times 10^{-19}\ \mathrm{J}$$

Thus, there are about $(1.25 \times 10^{-16}\ \mathrm{J}/3.97 \times 10^{-19}\ \mathrm{J}) = 315$ photons per pixel. Is this enough?

Example 2

1. *In the Eddington approximation the radiance field is expanded to first order as* $I(\mu) = I_0 + \mu I_1$ *(no azimuthal dependence). What is the upwelling and downwelling hemispheric flux, the net flux, and the actinic flux, in terms of* I_0 *and* I_1*?*

The upwelling flux is the cosine weighting of the upwelling radiance:

$$F^{\uparrow} = \int_0^{2\pi} \int_0^1 I(\mu, \phi)\, \mu \, d\mu \, d\phi \quad = \quad \int_0^{2\pi} \int_0^1 (I_0 + \mu I_1)\mu \, d\mu \, d\phi$$

$$F^{\uparrow} = 2\pi \left[I_0 \frac{1}{2}\mu^2 + I_1 \frac{1}{3}\mu^3 \right]_0^1 \quad = \pi I_0 + \frac{2\pi}{3} I_1$$

The downwelling flux is

$$F^{\downarrow} = \int_0^{2\pi} \int_{-1}^0 I(\mu, \phi)\, |\mu| \, d\mu \, d\phi \quad = \quad - \int_0^{2\pi} \int_0^1 (I_0 + \mu I_1)\mu \, d\mu \, d\phi$$

The cosine weighting factor must be positive so we assure this with the absolute value sign.

$$F^{\downarrow} = -2\pi \left[I_0 \frac{1}{2}\mu^2 + I_1 \frac{1}{3}\mu^3 \right]_{-1}^0 \quad = \pi I_0 - \frac{2\pi}{3} I_1$$

Note that if the radiation is isotropic, $I_1 = 0$, then the upwelling and downwelling flux are the same, namely πI_0.

The net flux is the difference between upwelling and downwelling hemispheric flux:

$$F_{net} = \int_0^{2\pi} \int_{-1}^{1} I(\mu, \phi)\, \mu \, d\mu \, d\phi \ = F^{\uparrow} - F^{\downarrow} \ = \frac{4\pi}{3} I_1$$

The net flux is proportional to I_1. The upwelling and downwelling hemispheric fluxes are positive, but the net flux may be positive or negative depending on whether the net energy flow is upwards or downwards.
The actinic flux is the integral of the radiance over all angles without the cosine weighting:

$$F_0 = \int_0^{2\pi} \int_{-1}^{1} I(\mu, \phi)\, d\mu \, d\phi \ = \int_0^{2\pi} \int_{-1}^{1} (I_0 + \mu I_1)\, d\mu \, d\phi$$

$$F_0 = 2\pi \left[I_0 \mu + I_1 \frac{1}{2} \mu^2 \right]_{-1}^{1} \ = 4\pi I_0$$

The actinic flux is proportional to I_0, which is the mean radiance.